Math 10B - Calculus of Several Variables II - Winter 2011

March 9, 2011

Practice Final

Name: ____

There is no need to use calculators on this exam. All electronic devices should be turned off and put away. The only things you are allowed to have are: a writing utensil(s) (pencil preferred), an eraser, and an exam. All answers should be given as exact, closed form numbers as opposed to decimal approximations (i.e. π as opposed to 3.14159265358979...). Cheating is strictly forbidden. You may leave when you are done. Good luck!

Problem	Score
1	/10
2	/10
3	/20
4	/20
5	/20
6	/20
7	/20
8	/10
9	/20
10	/20
Score	/170

Problem 1 (10 points)**.** *Compute the following integral:*

$$\int_0^{\frac{\pi}{2}} \int_y^{\frac{\pi}{2}} \sin x^2 \; dx dy.$$

Draw the region of integration.

Problem 2 (10 points)**.** *Find the volume of the region bounded by* $z = x^2+y^2-1$ *and* $z = 1-x^2-y^2$.

Problem 3 (20 points)**.**

(a) *(10 points) Compute the Jacobian* $\dfrac{\partial(x, y)}{\partial(r, \theta)}$ *for changing Cartesian coordinates to polar coordinates.*

(b) *(10 points) Let* D *be the region bounded by* $x^2 + y^2 = 5$ *where* $x \geq 0$*. Compute the integral*

$$\iint_D e^{x^2+y^2}\, dA.$$

Problem 4 (20 points)**.**

(a) *(10 points) Parametrize the circle of radius* r*.*

(b) *(10 points) Use this parametrization to show that the circumference of the circle of radius* r *is* $2\pi r$*. (Hint: Use arclength.)*

Problem 5 (20 points)**.** *Let C be the boundary of the region bounded by $y = x^2$ and $x = y^2$, oriented counterclockwise.*

(a) *(10 points) Compute the integral*

$$\int_C \arctan x^3 \, dx + \ln(y^2 + 1) \, dy.$$

(b) *(10 points) Compute the integral*

$$\int_C y \, dx - x \, dy.$$

Problem 6 (20 points)**.** *Determine whether the following vector fields are conservative. Find a scalar potential function for the ones that are conservative.*

(a) *(10 points)*

$$\vec{\mathbf{F}}(x, y) = (2x \sin y, x^2 \cos y).$$

(b) *(10 points)*

$$\vec{\mathbf{G}}(x, y, z) = (y + z, 2z, x + y).$$

Problem 7 (20 points)**.** *Let f be a $\mathcal{C}^1$ function on some region $D \subset \mathbb{R}^2$, and consider the surface given by $z = f(x, y)$. Show that the surface area of this surface is given by*

$$S.A. = \iint_D \sqrt{f_x^2 + f_y^2 + 1}\, dA.$$

Hint: Recall that surface area is given by

$$S.A. = \iint_{\mathbf{X}} dS$$

where $\mathbf{X}$ is a parametrization of the surface.

Problem 8 (10 points)**.** *Let S denote the closed cylinder with bottom given by $z = 0$, top given by $z = 7$, and lateral surface given by $x^2 + y^2 = 49$. Orient S with outward normals. Compute the following integral:*

$$\iint_S (-y\mathbf{i} + x\mathbf{j}) \cdot d\mathbf{S}.$$

Problem 9 (20 points)**.** *Let S be the sphere given by $x^2+y^2+z^2=1$ with outward pointing normals.*

(a) *(10 points) Let $\mathbf{F}(x,y,z)=(2xyz+5z, e^x\cos yz, x^2y)$. Compute*

$$\iint_S curl\,\mathbf{F}\cdot d\mathbf{S}.$$

(b) *(10 points) Let $\mathbf{G}(x,y,z)=(x,y,z)$. Compute*

$$\iint_S \mathbf{G}\cdot d\mathbf{S}.$$

Hint: The volume of a sphere of radius r is given by $V=\frac{4}{3}\pi r^3$.

Problem 10 (20 points)**.** *Verify that Stokes' theorem implies Green's theorem. Hint: Use the vector field* $\mathbf{F}(x, y, z) = (M(x, y), N(x, y), 0)$.